where

$$F_0 = \left\{J_0^2(K_1r_1) + J_1^2(K_1r_1) - \frac{2}{K_1r_1} J_1(K_1r_1)J_0(K_1r_1)\right\} \tag{17}$$

$$G_1 = K_1^2 + \beta_2^2 \tag{18}$$

$$G_2 = K_1^2 - \beta_2^2 \tag{19}$$

$$P = 4K_1^2J_0^2(K_1r_1) + \pi^2r_1^2F_1^2K_2^2\left[\{F_0 - J_1^2(K_1r_1)\}\{K_2^2\mu_r - K_1^2\} + \frac{2K_1}{r_1} J_1(K_1r_1)J_0(K_1r_1)\{1 - \mu_r\}\right] \tag{20}$$

$$F_1 = J_0(K_2r_1)Y_1(K_2mr_1) - Y_0(K_2r_1)J_1(K_2mr_1) \tag{21}$$

$$P_1 = K_1^2d + \frac{K_2^2}{2}\mu_rt - \psi_1d(K_1^2 + K_2^2) + K_1^2K_2^2\psi_1^2\frac{d(d+t)}{2\mu_r} \tag{22}$$

$$P_2 = \{K_2^2\mu_rt + (a - t)K_1^2\} - \phi_2\mu_rt(K_2^2 - K_1^2) + \phi_2^2t^2(K_1^2K_2^2)\{\mu_rt + \mu_r^2(a - t)\}. \tag{23}$$

Evaluation of $\tan_e \delta$ and $\tan_m \delta$

$\tan_e \delta$ and $\tan_m \delta$ are evaluated from measurements of Q for the two configurations. The procedure for evaluating $\tan_e \delta$ and $\tan_m \delta$ from these Q values is exactly the same as in the earlier techniques,[1,2] except that the resulting equations are a lot more complex and tedious to compute.

However, by choosing the total cavity length to be at least four half wavelengths, and if the Q of the cavity drops to at least 2/3 of its value by inserting the sample by half a wavelength, one may take certain approximations[2] which simplify the expressions very considerably and introduce an error of the order of 2 to 4 percent in the evaluation of $\tan_e \delta$ and $\tan_m \delta$. These expressions are given below. If Q_1 is the loaded Q for zero insertion, and Q_2 the loaded Q for an insertion by half a wavelength, one obtains

CONCLUSIONS

These techniques offer the possibility of very quick and accurate evaluation of permittivity and permeability of a broad range of specimen dimensions, both in the form of rods and slabs if the suitable charts are prepared. In addition, these are the only available techniques based on accurate theoretical solutions, in which one may evaluate all four parameters using only a single specimen, resulting in an enormous convenience.

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Design of TEM Equal Stub Admittance Filters

Filters formed in TEM transmission lines by short-circuited stubs that are $\lambda/4$ in length at the design center frequency and separated by the same length have useful bandpass properties in wideband (typically 10 percent to one-octave bandwidth) applications.

The usual design of a filter of this type is for a maximally flat or Chebyshev response, which requires a tapering of the characteristic admittances of the stubs [1]. The procedure described here, which may be used successfully in many applications, requires that the stub admittances all be equal. In those applications where maximum flatness or equal ripple are not required, this is a simple, inexpensive, and easily designed structure.

The theoretical approach described here is similar to that of Mumford [1], in which we first state the form of our filter and then analyze on the basis of the exact filter model. This approach very quickly gave us the insertion loss characteristics we were seeking.

Another approach to TEM filter synthesis is to use the Richards' transformation and the Kuroda identities [2]. Various "optimum" structures in the Butterworth or Chebyshev sense have been analyzed wherein a network is synthesized to approximate a desired function. This approach has not been necessary in this case.

The analytical expressions for insertion loss are derived for any number of resonators. The assumption of dissipationless filters was made; dissipation is negligible for the relatively wideband filters considered here. Curves are available for one to eight resonators, which enables a systematic design. Bandwidth, insertion loss, and characteristic admittance may be rigorously determined in specific applications. Examples of this are given. A bandpass filter is tested and the results are shown to agree with the predictions of the theory.

Using well-known techniques, a model is analyzed for the TEM structures considered. Figure 1 shows a form suitable for the purposes of our analysis. The resonators are considered lossless. The mathematical derivation is briefly outlined here.

The filter is considered lossless, linear, passive, reciprocal, and symmetrical. The insertion loss is given by

$$L = 10\log\left[1 - \frac{(B_N - C_N)^2}{4}\right],$$

where B_N and C_N are determined from

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^N = \begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix}.$$

This $ABCD$ matrix is for a single (line-stub-line) section. This is then multiplied N times using techniques described elsewhere [3].

The insertion loss is given by

$$L = 10\log\left\{1 + \frac{K^2q^2}{4(1-q^2)} P_N^2\left[2q\left(1 + \frac{K}{1}\right)\right]\right\}, \tag{1}$$

where

1) K = characteristic admittance of stub resonator normalized to line admittance,
2) N = number of stubs,
3) $q = \cos\theta = \cos(2\pi d/\lambda)$ where d is the stub length, and
4) P_N = Chebyshev polynomial of the second kind.

This expression has been plotted for $N=1$ through 8 with K as a parameter and q as the abscissa.[1] Curves for $N=3$, 4, and 7 are shown in Figs. 2, 3, and 4, respectively.

As far as these graphs are concerned, we may immediately state the following. We have $q=\cos\theta$, so q varies as $-1 \leq q \leq 1$ as θ or λ varies. At $q=0$, the insertion loss is zero for all N and K. At $q=\pm 1$, the insertion loss is infinity for all N and K. In addition, P_N^2 is an even function of q, so that it is only necessary to plot the region $0 \leq q \leq 1$ due to symmetry. All values of λ of interest are mapped in this region.

It is seen from (1) that, for $N=1$ and $N=2$, the quarter-wave shorted-stub filter is identical to the maximally flat filter of Mumford [1].

The next section gives design examples and insertion loss curves. With the use of (1), we may easily derive approximate equations for specific regions of the insertion loss characteristic. These are useful when a curve is not available. We do not reproduce these approximations, since there are so many depending on the region of interest.

As a design example let it be required to have a bandpass TEM filter with a minimum 3-dB bandwidth of 630 MHz or 70 percent, a center frequency of 900 MHz, and a minimum rejection of 20 dB at 900±500 MHz. Each of our stubs must be $\lambda/4$ in length or 8.33 cm at 900 MHz. The 70-percent bandwidth corresponds to a 3-dB frequency of $q=0.525$ and a 20-dB frequency of $q=0.766$. Examination of the curves show that a filter of $N=4$ and $K=1.4$ (35.7-ohm stubs) will do the job. The ripple will be 0.3 dB at one point and this will be adequate for many applications.

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[1] Complete graphs are available from ADI Auxiliary Publications Program.

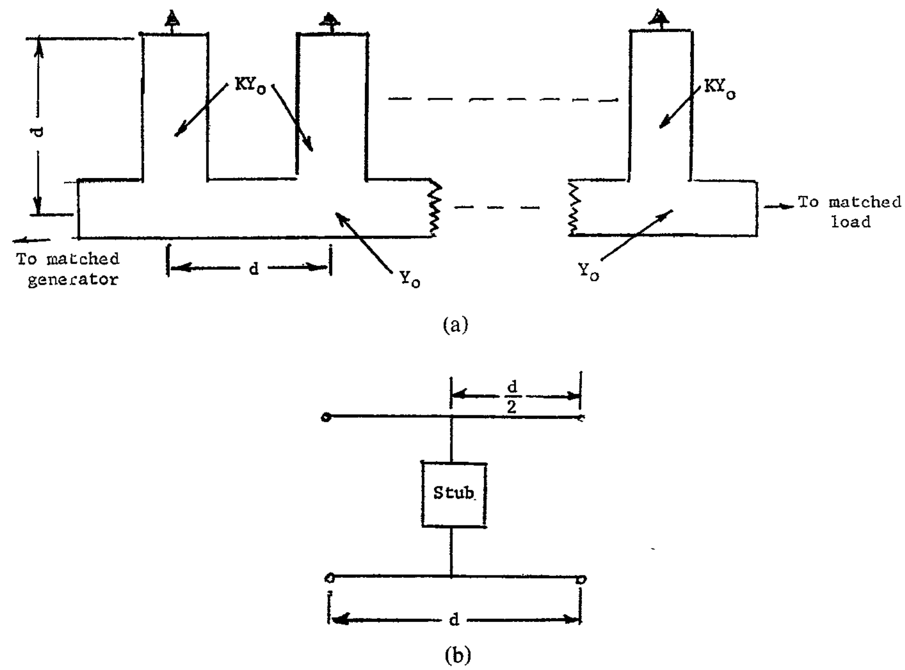

Fig. 1. Equal stub admittance bandpass filter structure. (a) Schematic. (b) Typical iterative section.

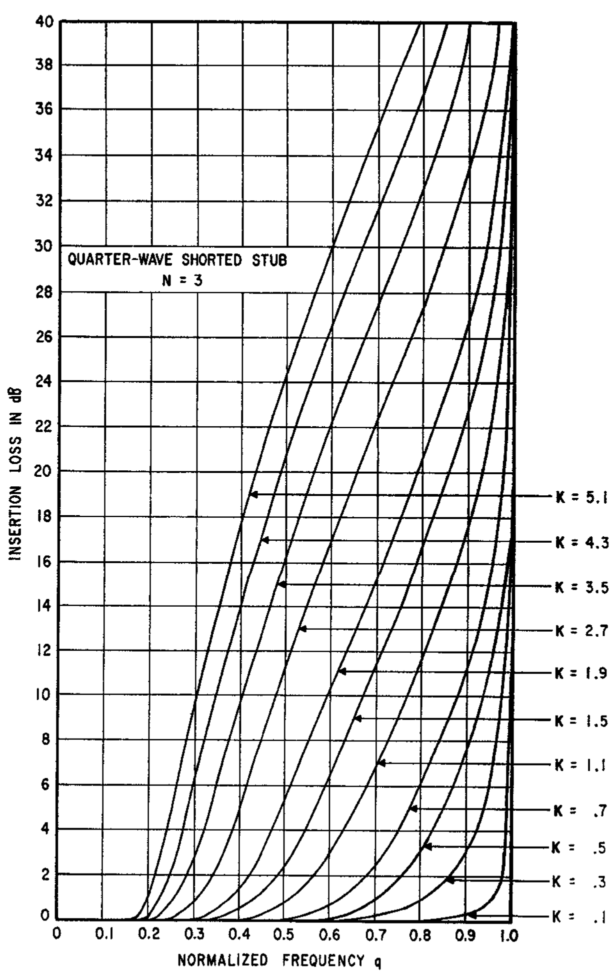

Fig. 2.

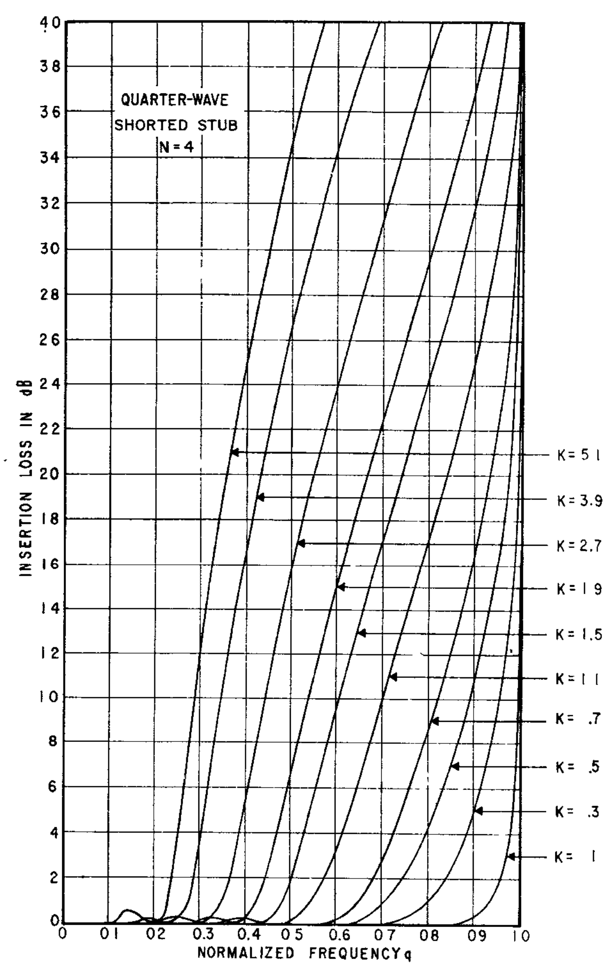

Fig. 3.

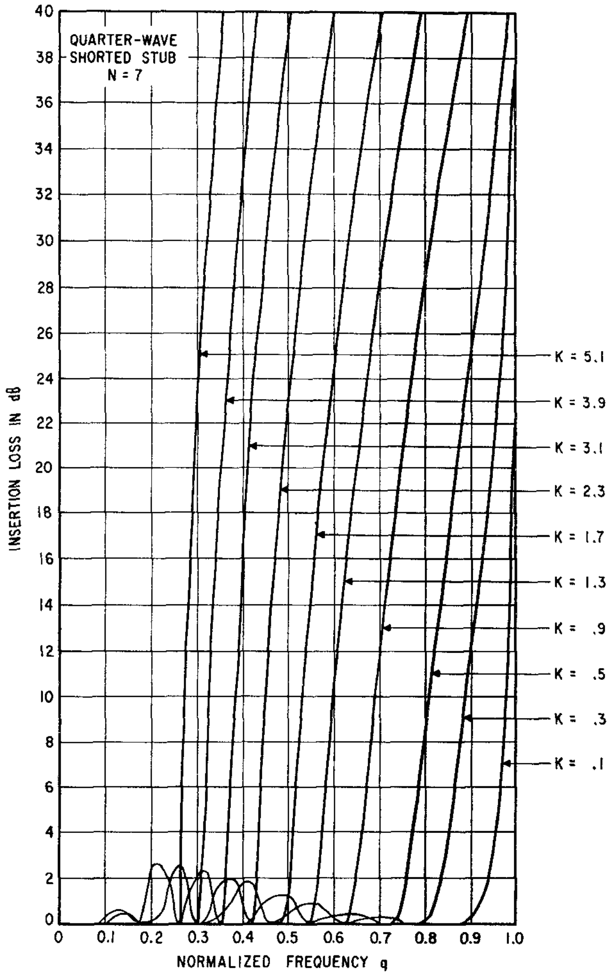

Fig. 4.

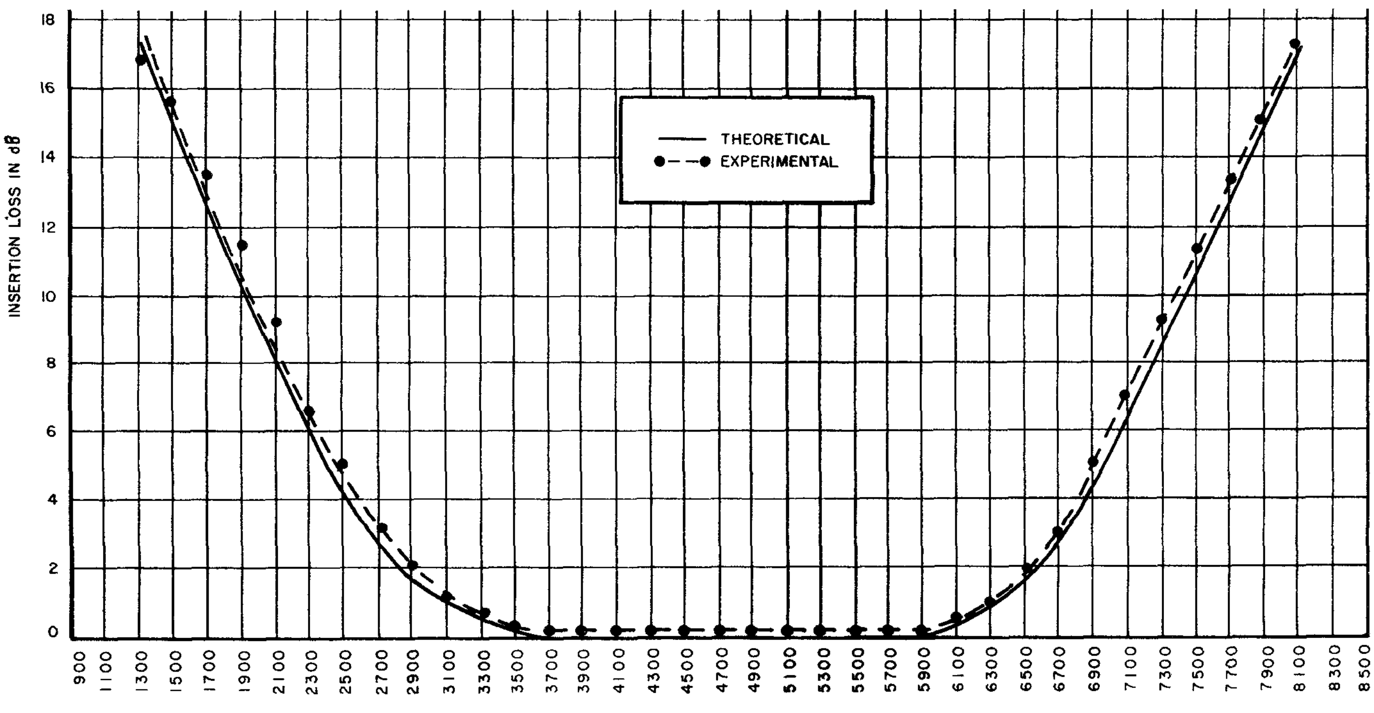

Fig. 5 Theoretical and experimental insertion loss of quarter-wave-coupled shorted-stub filter ($N=3$).

As a second example, consider a bandpass filter with a center frequency of 1000 MHz, a minimum 3-dB bandwidth of 100 percent, and a 20-dB rejection from at least 133 to 1867 MHz. The ripple is to be less than 0.05 dB.

We easily calculate the resonator length as 7.5 cm. The 3-dB frequency is calculated as $q=0.707$, and the 20-dB frequency as $q=0.98$. Examination of the curves shows that a three-resonator filter with $K=0.8$ (62.5-ohm stubs) will satisfy the requirements.

Many other possibilities are open. For example, the designer may examine the curves to see if the characteristics of his filter may be satisfied by the filters considered, and use this design instead of one which may be "overdesigned." It must be determined if maximally flat or equal ripple response is really required. The filters also have bandstop properties, depending on the definition of center frequency.

These filters are not "optimum" designs in the sense of minimum number of resonators, best skirt response, etc. They are easily designed devices which, when they can be used, are quite functional.

To verify the theoretical results, a three-resonator quarter-wave shorted-stub coaxial filter was tested. The normalized characteristic admittance of the stub (K) was unity. The filter center frequency was 4700 MHz and its 3-dB bandwidth was 6400 MHz. Experimental and theoretical curves are shown in Fig. 5. The agreement is very close. The midband loss of the filter tested was about 0.2 dB, and the zero insertion loss design was shown to be valid.

In conclusion, we may say that the insertion loss versus frequency characteristics have been calculated for a class of equal stub admittance filters. Analytical expressions have been derived for any number of resonators, and graphs of the results have been prepared for from one to eight resonators. It was shown how the information presented enables the systematic design of filters of this class on the basis of insertion loss. It was also shown how bandwidth, admittance, etc., may be rigorously determined for specific cases of interest. The insertion loss curves are presented along with information as to their use and other pertinent theoretical material. Experimental verification of the theory has been presented. Further work along these lines that could be considered would concern other stub and line lengths, loss, time delay, and the analysis of transient response.

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On the Design of Stepped Transmission-Line Transformers

Abstract—**The problem of matching a complex load impedance to a given transmission line using a series matching transformer is considered with a view to minimizing the transformer length and overall insertion loss. A graphical technique is presented that leads to a solution for the length and characteristic impedance of the transformer section. It is shown that this transformer reduces to the usual quarter-wave transformer for the particular case where the load is purely resistive. The performance of the transformer is also compared with the quarter-wave transformer for the case of a complex load impedance and a numerical example is given. It is shown that the design procedure is relatively simple and may lead to a significant reduction in the overall length and insertion loss of the matching section. While there is no significant improvement in the bandwidth for frequency-dependent loads, the proposed design still offers attractive features for matching transmitting antennas.**

Present techniques for matching a radio-frequency transmission line to a given complex load impedance are based on cancelling the input reactance (or susceptance) of the load, as seen from a pair of available terminals, and transforming the real part to that of the given line. Common examples that employ this conjugate matching technique are the open- and short-circuited parallel stubs and the quarter-wave series transformer. The design procedure for single and multiple sections of these devices have been amply discussed in the literature [1]–[3].

The purpose of this correspondence is to present a simple graphical method for designing a single transmission-line cable suitable for matching a complex load impedance at a single frequency. To show this we consider the situation in Fig. 1 where the matching section of unknown length d and characteristic impedance Z_0' is inserted between the load Z_L and the feed line. The input impedance Z_s at the junction of the two lines is given by the relation [1],

$$Z_s = Z_0' \frac{Z_L + Z_0' \tanh(\gamma d)}{Z_L' + Z_L \tanh(\gamma d)}, \tag{1}$$

where the propagation constant γ is the sum of the attenuation constant α and the phase constant β. Equating the real terms to R and the imaginary terms to zero, we obtain after some simplification:

$$Z_0^2 = r - \frac{x^2}{1 - r} \tag{2}$$

$$y^2 = \left(\frac{1 - r^2}{x}\right)\left(r - \frac{x^2}{1 - r}\right), \tag{3}$$

where $y = \tan(\beta d)$ for $\alpha \to 0$, $r = R_L/R$, $x = x_L/R$, $Z_0 = Z_0'/R$, and $\beta = 2\pi/\lambda$. Equation (3) may be rewritten in the more convenient form:

$$\frac{(1 - r)^2 r}{[r - (1 + y^2)]} + x^2 = 0. \tag{3a}$$

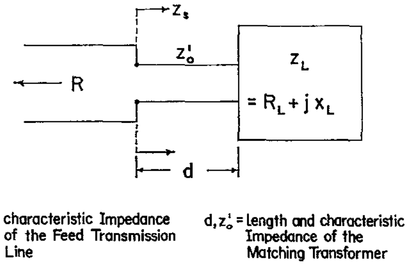

Fig. 1. Schematic diagram of the matched load Z_L.

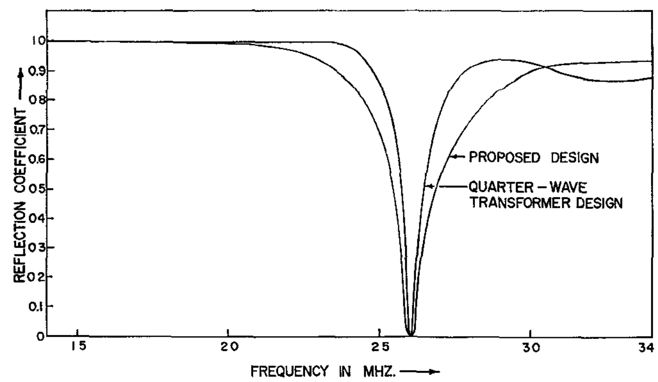

Fig. 2. Reflection coefficient versus frequency.

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